MATHEMATICAL PRACTICES
- MP.1 Make sense of problems and persevere in solving them.
- MP.2 Reason abstractly and quantitatively.
- MP.3 Construct viable arguments and critique the reasoning of others.
- MP.4 Model with mathematics.
- MP.5 Use appropriate tools strategically.
- MP.6 Attend to precision.
- MP.7 Look and make use of structure.
- MP.8 Look for and express regularity in repeated reasoning.

KINDERGARTEN
COUNTING AND CARDINALITY K.CC
- Know number names and the count sequence.
  - 2. Count forward beginning from a given number within the known sequence (instead of having to begin at 1).
  - 3. Write numbers from 0 to 20. Represent a number of objects with a written numeral 0-20 (with 0 representing a count of no objects).
- Count to tell the number of objects.
  - 4. Understand the relationship between numbers and quantities; connect counting to cardinality.
    - a. When counting objects, say the number names in the standard order, pairing each object with one and only one number name and each number name with one and only one object.
    - b. Understand that the last number name said tells the number of objects counted. The number of objects is the same regardless of their arrangement or the order in which they were counted.
    - c. Understand that each successive number name refers to a quantity that is one larger.
  - 5. Count to answer “how many?” questions about as many as 20 things arranged in a line, a rectangular array, or a circle, or as many as 10 things in a scattered configuration; given a number from 1–20, count out that many objects.
- Compare numbers.
  - 6. Identify whether the number of objects in one group is greater than, less than, or equal to the number of objects in another group, e.g., by using matching and counting strategies.
  - 7. Compare two numbers between 1 and 10 presented as written numerals.

OPERATIONS AND ALGEBRAIC THINKING K.OA
- Understand addition as putting together and adding to, and understand subtraction as taking apart and taking from.
  - 1. Represent addition and subtraction with objects, fingers, mental images, drawings, sounds (e.g., claps), acting out situations, verbal explanations, expressions, or equations.
  - 2. Solve addition and subtraction word problems, and add and subtract within 10, e.g., by using objects or drawings to represent the problem.
  - 3. Decompose numbers less than or equal to 10 into pairs in more than one way, e.g., by using objects or drawings, and record each decomposition by a drawing or equation (e.g., 5 = 2 + 3 and 5 = 4 + 1).
  - 4. For any number from 1 to 9, find the number that makes 10 when added to the given number, e.g., by using objects or drawings, and record the answer with a drawing or equation.
  - 5. Fluently add and subtract within 5.

MEASUREMENT AND DATA K.MD
- Describe and compare measurable attributes.
  - 1. Describe measurable attributes of objects, such as length or weight.
• Describe several measurable attributes of a single object.
  ▪ 2. Directly compare two objects with a measurable attribute in common, to see which object has “more of”/“less of” the attribute, and describe the difference. For example, directly compare the heights of two children and describe one child as taller/shorter.
• Classify objects and count the number of objects in each category.
  ▪ 3. Classify objects into given categories; count the numbers of objects in each category and sort the categories by count.

GEOMETRY K.G
• Identify and describe shapes (squares, circles, triangles, rectangles, hexagons, cubes, cones, cylinders, and spheres).
  ▪ 1. Describe objects in the environment using names of shapes, and describe the relative positions of these objects using terms such as above, below, beside, in front of, behind, and next to.
  ▪ 2. Correctly name shapes regardless of their orientations or overall size.
  ▪ 3. Identify shapes as two-dimensional (lying in a plane, “flat”) or three-dimensional (“solid”). Analyze, compare, create, and compose shapes.
  ▪ 4. Analyze and compare two- and three-dimensional shapes, in different sizes and orientations, using informal language to describe their similarities, differences, parts (e.g., number of sides and vertices/“corners”) and other attributes (e.g., having sides of equal length).
  ▪ 5. Model shapes in the world by building shapes from components (e.g., sticks and clay balls) and drawing shapes.
  ▪ 6. Compose simple shapes to form larger shapes. For example, “Can you join these two triangles with full sides touching to make a rectangle?”

FIRST GRADE
OPERATIONS AND ALGEBRAIC THINKING 1.OA
• Represent and solve problems involving addition and subtraction.
  ▪ 1. Use addition and subtraction within 20 to solve word problems involving situations of adding to, taking from, putting together, taking apart, and comparing, with unknowns in all positions, e.g., by using objects, drawings, and equations with a symbol for the unknown number to represent the problem.
  ▪ 2. Solve word problems that call for addition of three whole numbers whose sum is less than or equal to 20, e.g., by using objects, drawings, and equations with a symbol for the unknown number to represent the problem.
• Understand and apply properties of operations and the relationship between addition and subtraction.
  ▪ 3. Apply properties of operations as strategies to add and subtract. Examples: If 8 + 3 = 11 is known, then 3 + 8 = 11 is also known. (Commutative property of addition.) To add 2 + 6 + 4, the second two numbers can be added to make a ten, so 2 + 6 + 4 = 2 + 10 = 12. (Associative property of addition.)
  ▪ 4. Understand subtraction as an unknown-addend problem. For example, subtract 10 – 8 by finding the number that makes 10 when added to 8.
• Add and subtract within 20.
  ▪ 5. Relate counting to addition and subtraction (e.g., by counting on 2 to add 2).
  ▪ 6. Add and subtract within 20, demonstrating fluency for addition and subtraction within 10. Use strategies such as counting on; making ten (e.g., 8 + 6 = 8 + 2 + 4 = 10 + 4 = 14); decomposing a number leading to a ten (e.g., 13 – 4 = 13 – 3 – 1 = 10 – 1 = 9); using the relationship between addition and subtraction (e.g., knowing that 8 + 4 = 12, one knows 12 – 8 = 4); and creating equivalent but easier or known sums (e.g., adding 6 + 7 by creating the known equivalent 6 + 6 + 1 = 12 + 1 = 13).
• Work with addition and subtraction equations.
  ▪ 7. Understand the meaning of the equal sign, and determine if equations involving addition and subtraction are true or false. For example, which of the following equations are true and which are false? 6 = 6, 7 = 8 – 1, 5 + 2 = 2 + 5, 4 + 1 = 5 + 2.
  ▪ 8. Determine the unknown whole number in an addition or subtraction equation relating three whole numbers. For example, determine the unknown number that makes the equation true in each of the equations 8 + n? = 11, 5 = – 3, 6 + 6 = ?
MEASUREMENT AND DATA 1.MD

- Measure lengths indirectly and by iterating length units.
  - 1. Order three objects by length; compare the lengths of two objects indirectly by using a third object.
  - 2. Express the length of an object as a whole number of length units, by laying multiple copies of a shorter object (the length unit) end to end; understand that the length measurement of an object is the number of same-size length units that span it with no gaps or overlaps. Limit to contexts where the object being measured is spanned by a whole number of length units with no gaps or overlaps.
- Tell and write time.
  - 3. Tell and write time in hours and half-hours using analog and digital clocks.
- Represent and interpret data.
  - 4. Organize, represent, and interpret data with up to three categories; ask and answer questions about the total number of data points, how many in each category, and how many more or less are in one category than in another.

GEOMETRY 1.G

- Reason with shapes and their attributes.
  - 1. Distinguish between defining attributes (e.g., triangles are closed and three-sided) versus non-defining attributes (e.g., color, orientation, overall size); build and draw shapes to possess defining attributes.
  - 2. Compose two-dimensional shapes (rectangles, squares, trapezoids, triangles, half-circles, and quarter-circles) or three-dimensional shapes (cubes, right rectangular prisms, right circular cones, and right circular cylinders) to create a composite shape, and compose new shapes from the composite shape.
  - 3. Partition circles and rectangles into two and four equal shares, describe the shares using the words halves, fourths, and quarters, and use the phrases half of, fourth of, and quarter of. Describe the whole as two of, or four of the shares. Understand for these examples that decomposing into more equal shares creates smaller shares.

SECOND GRADE

OPERATIONS AND ALGEBRAIC THINKING 2.OA

- Represent and solve problems involving addition and subtraction.
  - 1. Use addition and subtraction within 100 to solve one- and two-step word problems involving situations of adding to, taking from, putting together, taking apart, and comparing, with unknowns in all positions, e.g., by using drawings and equations with a symbol for the unknown number to represent the problem.1
- Add and subtract within 20.
  - 2. Fluently add and subtract within 20 using mental strategies.2 By end of Grade 2, know from memory all sums of two one-digit numbers.
- Work with equal groups of objects to gain foundations for multiplication.
  - 3. Determine whether a group of objects (up to 20) has an odd or even number of members, e.g., by pairing objects or counting them by 2s; write an equation to express an even number as a sum of two equal addends.

NUMBER AND OPERATIONS IN BASE TEN 2.NBT

- Understand place value.
  - 2. Count within 1000; skip-count by 5s, 10s, and 100s.

MEASUREMENT AND DATA 2.MD

- Measure and estimate lengths in standard units.
  - 1. Measure the length of an object by selecting and using appropriate tools such as rulers, yardsticks, meter sticks, and measuring tapes.
  - 2. Measure the length of an object twice, using length units of different lengths for the two measurements; describe how the two measurements relate to the size of the unit chosen.
  - 4. Measure to determine how much longer one object is than another, expressing the length difference in terms of a standard length unit.
- Relate addition and subtraction to length.
  - 5. Use addition and subtraction within 100 to solve word problems involving lengths that are given in the same units, e.g., by using drawings (such as drawings of rulers) and equations with a symbol for the unknown number to
represent the problem.

- 6. Represent whole numbers as lengths from 0 on a number line diagram with equally spaced points corresponding to the numbers 0, 1, 2, ..., and represent whole-number sums and differences within 100 on a number line diagram.

- 9. Generate measurement data by measuring lengths of several objects to the nearest whole unit, or by making repeated measurements of the same object. Show the measurements by making a line plot, where the horizontal scale is marked off in whole-number units.

- 10. Draw a picture graph and a bar graph (with single-unit scale) to represent a data set with up to four categories. Solve simple put together, take-apart, and compare problems using information presented in a bar graph.

GEOMETRY 2.G

- Reason with shapes and their attributes.

  - 1. Recognize and draw shapes having specified attributes, such as a given number of angles or a given number of equal faces. Identify triangles, quadrilaterals, pentagons, hexagons, and cubes.

  - 2. Partition a rectangle into rows and columns of same-size squares and count to find the total number of them.

  - 3. Partition circles and rectangles into two, three, or four equal shares, describe the shares using the words halves, thirds, half of, a third of, etc., and describe the whole as two halves, three thirds, four fourths. Recognize that equal shares of identical wholes need not have the same shape.

THIRD GRADE

OPERATIONS AND ALGEBRAIC THINKING 3.OA

- Represent and solve problems involving multiplication and division.

  - 3. Use multiplication and division within 100 to solve word problems in situations involving equal groups, arrays, and measurement quantities, e.g., by using drawings and equations with a symbol for the unknown number to represent the problem.

  - 4. Determine the unknown whole number in a multiplication or division equation relating three whole numbers. For example, determine the unknown number that makes the equation true in each of the equations $8 \times ? = 48$, $5 = \div 3$, $6 \times 6 = ?$.

  - 5. Apply properties of operations as strategies to multiply and divide. Examples: If $6 \times 4 = 24$ is known, then $4 \times 6 = 24$ is also known. (Commutative property of multiplication.) $3 \times 5 \times 2$ can be found by $3 \times 5 = 15$, then $15 \times 2 = 30$, or by $5 \times 2 = 10$, then $3 \times 10 = 30$. (Associative property of multiplication.) Knowing that $8 \times 5 = 40$ and $8 \times 2 = 16$, one can find $8 \times 7$ as $8 \times (5 + 2) = (8 \times 5) + (8 \times 2) = 40 + 16 = 56$. (Distributive property.)

  - 6. Understand division as an unknown-factor problem. For example, find $32 \div 8$ by finding the number that makes 32 when multiplied by 8.

  - 9. Identify arithmetic patterns (including patterns in the addition table or multiplication table), and explain them using properties of operations. For example, observe that 4 times a number is always even, and explain why 4 times a number can be decomposed into two equal addends.

MEASUREMENT AND DATA 3.MD

- Solve problems involving measurement and estimation of intervals of time, liquid volumes, and masses of objects.

  - 1. Tell and write time to the nearest minute and measure time intervals in minutes. Solve word problems involving addition and subtraction of time intervals in minutes, e.g., by representing the problem on a number line diagram.

  - 4. Generate measurement data by measuring lengths using rulers marked with halves and fourths of an inch. Show the data by making a line plot, where the horizontal scale is marked off in appropriate units — whole numbers, halves, or quarters.

- Represent and interpret data.

  - 4. Generate measurement data by measuring lengths using rulers marked with halves and fourths of an inch. Show the data by making a line plot, where the horizontal scale is marked off in appropriate units — whole numbers, halves, or quarters.

- Geometric measurement: understand concepts of area and relate area to multiplication and to addition.

  - 5. Recognize area as an attribute of plane figures and understand concepts of area measurement.
    - a. A square with side length 1 unit, called “a unit square,” is said to have “one square unit” of area, and can be used to measure area.
b. A plane figure which can be covered without gaps or overlaps by \( n \) unit squares is said to have an area of \( n \) square units.

- Measure areas by counting unit squares (square cm, square m, square in, square ft, and improvised units).
- Relate area to the operations of multiplication and addition.
  - Find the area of a rectangle with whole-number side lengths by tiling it, and show that the area is the same as would be found by multiplying the side lengths.
  - Recognize area as additive. Find areas of rectilinear figures by decomposing them into non-overlapping rectangles and adding the areas.

NUMBER AND OPERATIONS—FRACTIONS 3.NF
- Develop understanding of fractions as numbers.
  - Understand a fraction \( \frac{1}{b} \) as the quantity formed by 1 part when a whole is partitioned into \( b \) equal parts; understand a fraction \( \frac{a}{b} \) as the quantity formed by \( a \) parts of size \( \frac{1}{b} \).
  - Compare two fractions with the same numerator or the same denominator by reasoning about their size. Recognize that comparisons are valid only when the two fractions refer to the same whole. Record the results of comparisons with the symbols >, =, or <, and justify the conclusions, e.g., by using a visual fraction model.
- Explain equivalence of fractions in special cases, and compare fractions by reasoning about their size.
  - Understand two fractions as equivalent (equal) if they are the same size, or the same point on a number line.
  - Recognize and generate simple equivalent fractions, e.g., \( \frac{1}{2} = \frac{2}{4}, \frac{4}{6} = \frac{2}{3} \). Explain why the fractions are equivalent, e.g., by using a visual fraction model.

GEOMETRY 3.G
- Reason with shapes and their attributes.
  - Understand that shapes in different categories (e.g., rhombuses, rectangles, and others) may share attributes (e.g., having four sides), and that the shared attributes can define a larger category (e.g., quadrilaterals). Recognize rhombuses, rectangles, and squares as examples of quadrilaterals, and draw examples of quadrilaterals that do not belong to any of these subcategories.
  - Partition shapes into parts with equal areas. Express the area of each part as a unit fraction of the whole. For example, partition a shape into 4 parts with equal area, and describe the area of each part as \( \frac{1}{4} \) of the area of the shape.

FOURTH GRADE

OPERATIONS AND ALGEBRAIC THINKING 4.OA
- Use the four operations with whole numbers to solve problems.
  - Interpret a multiplication equation as a comparison, e.g., interpret \( 35 = 5 \times 7 \) as a statement that 35 is 5 times as many as 7 and 7 times as many as 5. Represent verbal statements of multiplicative comparisons as multiplication equations.
  - Multiply or divide to solve word problems involving multiplicative comparison, e.g., by using drawings and equations with a symbol for the unknown number to represent the problem, distinguishing multiplicative comparison from additive comparison.
- Generate and analyze patterns.
  - Generate a number or shape pattern that follows a given rule. Identify apparent features of the pattern that were not explicit in the rule itself. For example, given the rule “Add 3” and the starting number 1, generate terms in the resulting sequence and observe that the terms appear to alternate between odd and even numbers. Explain informally why the numbers will continue to alternate in this way.

MEASUREMENT AND DATA 4.MD
- Represent and interpret data.
  - Make a line plot to display a data set of measurements in fractions of a unit (1/2, 1/4, 1/8). Solve problems involving addition and subtraction of fractions by using information presented in line plots. For example, from a line plot find and interpret the difference in length between the longest and shortest specimens in an insect collection.

NUMBER AND OPERATIONS IN BASE TEN 4.NBT
- Generalize place value understanding for multi-digit whole numbers.
  - 1. Recognize that in a multi-digit whole number, a digit in one place represents ten times what it represents in the place to its right. For example, recognize that 700 ÷ 70 = 10 by applying concepts of place value and division.
  - 2. Read and write multi-digit whole numbers using base-ten numerals, number names, and expanded form. Compare two multi-digit numbers based on meanings of the digits in each place, using >, =, and < symbols to record the results of comparisons.
  - 3. Use place value understanding to round multi-digit whole numbers to any place.
- Use place value understanding and properties of operations to perform multi-digit arithmetic.
  - 4. Fluently add and subtract multi-digit whole numbers using the standard algorithm.

**NUMBER AND OPERATIONS—FRACTIONS 3.4.NF**
- Extend understanding of fraction equivalence and ordering.
  - 1. Explain why a fraction a/b is equivalent to a fraction (n × a)/(n × b) by using visual fraction models, with attention to how the number and size of the parts differ even though the two fractions themselves are the same size. Use this principle to recognize and generate equivalent fractions.
- Build fractions from unit fractions by applying and extending previous understandings of operations on whole numbers.
  - 3. Understand a fraction a/b with a > 1 as a sum of fractions 1/b.
    - a. Understand addition and subtraction of fractions as joining and separating parts referring to the same whole.
    - b. Decompose a fraction into a sum of fractions with the same denominator in more than one way, recording each decomposition by an equation. Justify decompositions, e.g., by using a visual fraction model. Examples: 3/8 = 1/8 + 1/8 + 1/8; 3/8 = 1/8 + 2/8; 2 1/8 = 1 + 1 + 1/8 = 8/8 + 8/8 + 1/8.
    - d. Solve word problems involving addition and subtraction of fractions referring to the same whole and having like denominators, e.g., by using visual fraction models and equations to represent the problem.
- Extend understanding of fraction equivalence and ordering.
  - 7. Compare two decimals to hundredths by reasoning about their size. Recognize that comparisons are valid only when the two decimals refer to the same whole.

**GEOMETRY 4.G**
- Draw and identify lines and angles, and classify shapes by properties of their lines and angles.
  - 1. Draw points, lines, line segments, rays, angles (right, acute, obtuse), and perpendicular and parallel lines. Identify these in two-dimensional figures.
  - 3. Recognize a line of symmetry for a two-dimensional figure as a line across the figure such that the figure can be folded along the line into matching parts. Identify line-symmetric figures and draw lines of symmetry.

**FIFTH GRADE**

**OPERATIONS AND ALGEBRAIC THINKING 5.OA**
- Analyze patterns and relationships.
  - 3. Generate two numerical patterns using two given rules. Identify apparent relationships between corresponding terms. Form ordered pairs consisting of corresponding terms from the two patterns, and graph the ordered pairs on a coordinate plane. For example, given the rule “Add 3” and the starting number 0, and given the rule “Add 6” and the starting number 0, generate terms in the resulting sequences, and observe that the terms in one sequence are twice the corresponding terms in the other sequence. Explain informally why this is so.

**NUMBER AND OPERATIONS IN BASE TEN 5.NBT**
- Understand the place value system.
  - 1. Recognize that in a multi-digit number, a digit in one place represents 10 times as much as it represents in the place to its right and 1/10 of what it represents in the place to its left.
- Perform operations with multi-digit whole numbers and with decimals to hundredths.
  - 5. Fluently multiply multi-digit whole numbers using the standard algorithm.
  - 6. Find whole-number quotients of whole numbers with up to four-digit dividends and two-digit divisors, using strategies based on place value, the properties of operations, and/or the relationship between multiplication and division. Illustrate and explain the calculation by using equations, rectangular arrays, and/or area models.
7. Add, subtract, multiply, and divide decimals to hundredths, using concrete models or drawings and strategies based on place value, properties of operations, and/or the relationship between addition and subtraction; relate the strategy to a written method and explain the reasoning used.

**NUMBER AND OPERATIONS—FRACTIONS 5.NF**

- Use equivalent fractions as a strategy to add and subtract fractions.
- Apply and extend previous understandings of division to divide unit fractions by whole numbers and whole numbers by unit fractions.1
  - a. Interpret division of a unit fraction by a non-zero whole number, and compute such quotients. For example, create a story context for \( \frac{1}{3} \div 4 \), and use a visual fraction model to show the quotient.

**GEOMETRY 5.G**

- Graph points on the coordinate plane to solve real-world and mathematical problems.
  - 3. Understand that attributes belonging to a category of two-dimensional figures also belong to all subcategories of that category. For example, all rectangles have four right angles and squares are rectangles, so all squares have four right angles.

**SIXTH GRADE**

**THE NUMBER SYSTEM 6.NS**

- Apply and extend previous understandings of numbers to the system of rational numbers.
  - 5. Understand that positive and negative numbers are used together to describe quantities having opposite directions or values (e.g., temperature above/below zero, elevation above/below sea level, credits/debits, positive/negative electric charge); use positive and negative numbers to represent quantities in real-world contexts, explaining the meaning of 0 in each situation.
  - 6. Understand a rational number as a point on the number line. Extend number line diagrams and coordinate axes familiar from previous grades to represent points on the line and in the plane with negative number coordinates.
    - a. Recognize opposite signs of numbers as indicating locations on opposite sides of 0 on the number line; recognize that the opposite of the opposite of a number is the number itself, e.g., \(-(-3) = 3\), and that 0 is its own opposite.
    - b. Understand signs of numbers in ordered pairs as indicating locations in quadrants of the coordinate plane; recognize that when two ordered pairs differ only by signs, the locations of the points are related by reflections across one or both axes.
    - c. Find and position integers and other rational numbers on a horizontal or vertical number line diagram; find and position pairs of integers and other rational numbers on a coordinate plane.
  - 7. Understand ordering and absolute value of rational numbers.
    - a. Interpret statements of inequality as statements about the relative position of two numbers on a number line diagram. For example, interpret \(-3 > -7\) as a statement that \(-3\) is located to the right of \(-7\) on a number line oriented from left to right.

**RATIOS AND PROPORTIONAL RELATIONSHIPS 6.RP**

- Understand ratio concepts and use ratio reasoning to solve problems.
  - 1. Understand the concept of a ratio and use ratio language to describe a ratio relationship between two quantities. For example, “The ratio of wings to beaks in the bird house at the zoo was 2:1, because for every 2 wings there was 1 beak.” “For every vote candidate A received, candidate C received nearly three votes.”.

**STATISTICS AND PROBABILITY 6.SP**

- Develop understanding of statistical variability.
  - 1. Recognize a statistical question as one that anticipates variability in the data related to the question and accounts for it in the answers. For example, “How old am I?” is not a statistical question, but “How old are the students in my school?” is a statistical question because one anticipates variability in students’ ages.
  - Summarize and describe distributions.
    - 4. Display numerical data in plots on a number line, including dot plots, histograms, and box plots.
    - 5. Summarize numerical data sets in relation to their context, such as by:
• a. Reporting the number of observations.
• b. Describing the nature of the attribute under investigation, including how it was measured and its units of measurement.